PLATOON-OPERATED STATIONS FOR QUASI-SYNCHRONOUS PRT NETWORKS

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by

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Abstract: A computer simulation of vehicle operations in an off-line, single-ramp personal rapid transit station is presented. Previous to entering the station, the vehicles form platoons in a queue area. The vehicles are assumed to leave the station in platoons and enter another queue area to await open slots for merging onto the main line. The independent variables of the simulation are the station throughput, flow rate on the main line, number of vehicle slots in the queue areas, and number of slots in the station. The dependent variables are the abort rate and time delays in the queue area. The main results of interest to station designers are curves relating throughput to the total number of slots in the station and queue areas. Curves corresponding to .5% and 1% abort rates are exhibited.

Notation

N	total number of vehicle slots in queue areas and station
N_F	number of slots in front queue
N_R	number of slots in rear queue
N_S	number of slots in station
a	vehicle acceleration, ft/sec^2
J	vehicle jerk, ft/sec^3
L	vehicle length, feet
T_D	station dwell time, seconds
T_S	platoon shift time, seconds
T	minimum elapsed time between platoon shifts, seconds
H	vehicle headway time, seconds
p	slot occupancy probability downstream or upstream of station
P	vehicle flow rate downstream or upstream of station, vehicles per hour
r	slot occupancy probability on station bypass track AC, Figure 1
R	vehicle flow rate on station bypass track AC, Figure 1
q	slot occupancy probability on station entrance or exit ramps
Q	station throughput, vehicles per hour
Q_{max}	upper bound on station capacity, vehicles per hour
A_r	abort rate
τ_q	average passenger time delay in vehicle queues, seconds
N_{AB}	number of slots between A and B, Figure 1
N_{BC}	number of slots between B and C, Figure 1
N_{AC}	number of slots between A and C, Figure 1
N_{DC}	number of slots between D and C, Figure 1

Introduction:

In recent years there has been an increasing amount of research and development on personal rapid transit (PRT) and dual-mode transit (DMT) systems. The systems feature transportation service on exclusive guideways by computer-managed vehicles operating under automatic control. All stations are off-line stations. Modal split studies by Hamilton and Nance (1969), Bush (1972), Dais and Kornhauser (1973) and the University of Minnesota Task Force (1973) suggest that large-scale networks have the potential to attract a significant portion of urban trips. PRT vehicles are captive to the system and passengers gain access to the system by boarding vehicles at stations. DMT systems would also offer the transit service by captive vehicles. Additionally the guideways would be shared by vehicles which are manually operated on conventional streets but are automatically controlled and computer-managed when on the exclusive guideway system. The present paper focuses on the management and simulation of captive vehicles in a PRT or DMT system.

Three competing operating strategies which have been proposed for automatically controlled networks are synchronous, quasi-synchronous and asynchronous. Considerable controversy exists as to which is most advantageous. Demag-Messerschmidt (1971) has opted for the asynchronous approach. The quasi-synchronous approach has been studied by the Royal Aircraft Establishment (1969) and Godfrey (1968) who reported on queuing analyses at vehicle interchanges. These works show that interchanges can handle large throughputs with relatively small abort rates. Munson (1972) reported a simulation analysis for quasi-synchronous interchanges. York (1973) has simulated an entire quasi-synchronous PRT network. The present paper will report on the simulation of a station for a quasi-synchronous network.

The station simulated in the present paper is the single-ramp station shown in Figure 1. A single-ramp station will be easier to superimpose on the existing urban structure than a multi-ramp station. Modal split studies by the University of Minnesota Task Force (1973) and Dais and Kornhauser (1973) indicate that few, if any, stations in a large network would require vehicle throughputs of more than 1000 vehicles per hour. In fact, the peak-hour demands at most stations would be significantly less. The present paper will show that such throughputs are readily achievable in a single-ramp station.

The problem that is posed assumes that the main line of Figure 1 is operated synchronously. Vehicles enter and leave the station in platoons. A platoon leaving the station enters the front queue. Vehicles from the front queue merge to the main line as slots become available. Vehicles leaving the main line enter the rear queue area and form into platoons. Should the rear queue area be filled, then vehicles would be denied station access and would require rerouting. This situation is termed an abort. The primary burden of the present paper is to obtain numerical relationships between the abort rate, station throughput, main line loading, number of vehicle slots in the station, and number of slots in the front and rear queue areas.

Previous work on station operations was reported by Dais (1972), Bergren (1971), Wilson et.al. (1971), Munson (1972) and Bergmann (1972). Dais (1972) considered the platoon-operated station of Figure 1 and obtained a formula for the throughput. The analysis did not include the merge process and so the throughput formula provides an upper bound for the more completely formulated problem. Munson (1972) analyzed a belt-operated station. Belt-operated stations can provide high throughputs should this be required in some application. However, they would be more complex and costly then the stationary-platform alternative. Bergren(1971) considered a side-shift station which is possible with air-suspended vehicles and obtained an upper bound for the throughput. The analysis did not consider merging. From the standpoint of passenger time delays, throughput and abort rate, the side-shift station does not appear to be advantageous. Wilson et.al. (1971) performed a queuing analysis for single and multi-ramp stations. Their operational strategy did not incorporate vehicle queue areas. Consequently their analysis indicated high abort rates. Bergmann (1972) obtained an upper bound for a single-ramp station by assuming that vehicles would arrive cyclically. Consecutive vehicles would be separated by a headway time determined by allowable vehicle separations in the station approach. The analysis stopped short of considering the merge problem.

Statement of the Problem

The sketch in Figure 1 will facilitate the statement of the problem. The point C defines a merge slot and D is the front slot of the front queue, separated from C by N_{DC} slots. B indicates that slot on the main line defined by $N_{BC} = N_{DC}$. A defines the demerge slot and it follows that

$$N_{AC} = N_{AB} + N_{BC}. \tag{1}$$

The problem simulated assumes a station throughput of Q vehicles per hour and a flow rate of P vehicles per hour upstream of A and downstream of C. It follows that the flow-rate R on the main line between A and B is given by

$$R = P - Q. \tag{2}$$

The respective slot occupancy probabilities would be given by

$$\begin{aligned} p &= PH/3600 \\ q &= QH/3600 \\ r &= RH/3600, \end{aligned} \tag{3}$$

where H is the headway time in seconds. Here we assume that vehicles are randomly distributed in the slots along the main line.

We will next present a detailed account of the vehicle maneuvers in the queue areas and station. The account will be sufficiently detailed to permit the operational strategy to be simulated on a digital computer. The vehicles in the front queue, station and rear queue will be considered separately.

(1) Vehicles in the front queue always occupy the frontmost positions. At any time as many as N_F vehicles could occupy the front queue. At every headway-time step in the simulation a random number is generated to determine whether a vehicle occupies the slot B. The probability of the slot being occupied at any time is r. If the slot B is occupied then none of the vehicles in the queue advance. If the slot is not occupied, then the lead vehicle is discharged from the queue and the remaining vehicles are advanced one slot in the queue.

(2) Vehicles in the station always occupy the frontmost positions in the station. At any time, as many as N_S vehicles can occupy the station. The situation simulated is one for which passengers have been instructed to board the frontmost unoccupied vehicle in the station area. To be eligible for dispatching, a vehicle must either be occupied, be an empty called by some other station, or an empty which must be dispatched to make room for vehicles entering from the rear queue. All vehicles eligible for dispatch from the station are shifted in platoon fashion to the front queue areas. Those ineligible for dispatch would simultaneously be shifted to occupy the frontmost slots in the station. In order for a platoon to be shifted, two conditions must be satisfied:

a) At least T seconds must have elapsed since the previous platoon was shifted. T is defined by the equation

$$T = T_D + T_S , \qquad (4)$$

where T_D is the station dwell time and T_S is the time required to shift vehicles through a distance of N_S slots. It was shown previously by Dais (1972) that if the vehicles follow a maneuver profile as shown in Figure 2, then T_S is given by

$$T_S = a/J + \sqrt{a^2/J^2 + 4N_S L/a}. \qquad (5)$$

b) The front queue must have enough empty slots to contain the entire platoon to be dispatched. If conditions a) and b) are satisfied, the platoon is shifted. Otherwise the platoon would not advance. In this case, conditions a) and b) are checked at every headway-time step until a) and b) are satisfied.

3. Vehicles in the rear queue always occupy the frontmost positions. As many as N_R vehicles can occupy the rear queue at any time. At the same time that the station platoon is shifted, then all vehicles in the rear queue advance N_S slots. If the rear queue contains less than $N_S + 1$ vehicles, then all the vehicles will advance to the station. Otherwise N_S vehicles will advance to the station and the remainder will advance to the front of the queue.

At every headway-time step in the simulation a random number is generated to determine whether the demerge slot A is occupied. The probability that this slot is occupied is q. If the slot is occupied, a vehicle is added to the rear queue, provided that the queue contains less than N_R vehicles. Otherwise, an abort is tabulated. The simulation program counts the number of aborts and also calculates the average time that a vehicle spends in the queue.

To recapitulate, the mathematical problem that we have simulated is the following: Given the queue sizes N_F and N_R, the station size N_S, the throughput q and the station bypass flow rate $r = p - q$, find the abort rate A_r and average time delay in the queue areas, τ_q. Before proceeding to the numerical example of the next section, however, it is worth making a few observations. First, it would be pointless to consider queue areas where $N_F < N_S$ or $N_R < N_S$. In the former case it would be impossible to dispatch a platoon of N_S vehicles into the front queue. In the latter a platoon of N_S vehicles could not enter the station. Second, it is instructive to obtain an upper bound formula for Q_{max}, the maximum possible throughput in vehicles per hour. Since a platoon can pass through a station at most every T seconds, it follows that the throughput in platoons per hour is given by 3600/T. Since there can be at most N_S vehicles per platoon, it follows that

$$Q_{max} = 3600N_S/T = 3600\, q_{max}/H. \tag{6}$$

Third, it appears that the rear queue should be at least as large and usually larger than the front queue. Vehicles enter both queues at an average rate q. The service rate of a nonempty front queue is $1 - r$, whereas the average service rate of a nonempty rear queue is q_{max}. For the situations dealt with in the paper, $1 - r > q_{max}$. Furthermore, congestion in the front queue does not necessarily result in an abort, whereas congestion in the rear queue does.

A Numerical Example

In this section we begin a numerical example by assuming the parameter values in Table 1. The half-second headway requirement has been found necessary in application studies by Bush (1972) and Dais and Kornhauser (1973). The acceleration and jerk levels chosen would require seated passengers. The vehicle length chosen is approximately that of an auto. The 15 seconds for station dwell time is based on observations by University of Minnesota students of people leaving and entering elevators. To achieve a load-unload time of 15 seconds will require passive restraint mechanisms and automatic doors on vehicles, as well as careful control of passenger movements in the station.

All of the results presented in this section were obtained by setting $N_F = N_R$. Figures 3 and 4 present respectively curves of Abort Rate and Average Queue Delay Time for $N_S = 1, 3, 5$ and 9. The dashed lines indicate the upper bound on station throughput obtained from (6). Figure 5 shows N, the total queue and station slot requirement required to achieve a 1% abort rate with $p = .85$. That value was chosen as a design value since work by Godfrey (1968), Royal Aircraft Establishment (1969) and York (1973) indicates that larger values make merge management at interchanges difficult. The slot requirement N is given by

$$N = N_F + N_S + N_R. \tag{7}$$

The particular combinations of N_F and $N_S = N_R$ which minimize N for a given throughput were found by trial and error and are exhibited in Table 2. Some of those combinations were used to obtain the curves in Figures 3 and 4.

The curves in Figures 3 and 4 were obtained specifying N_F, N_R, N_S and P for several values of q in the range $0 < q < q_{max}$. Data points for A_r and τ_q were found on the basis of simulating 10000 time steps. The curves shown in Figure 3, 4, 5 and 6 were then obtained as a least squares fit to the data points. The curve in Figure 5 was obtained by taking the .5% and 1% cutoffs of curves like those in Figure 3.

Simulation results for abort rate versus throughput exhibited a certain degree of scatter. For this reason, curves obtained from a least squares fit were compared with results obtained from an analytical model of the problem. In this analysis, it was assumed that delay times caused by conflicts in the front queue area were negligible, so that each group of vehicles was processed in exactly T seconds.

If $P_n(T)$ represents the probability that there are n vehicles in the rear queue at the end of the cycle of T seconds, and $P_n(o)$ represents the probability of n vehicles in the rear queue at the beginning of the cycle, then the equations governing the rear queue are

$$\left.\begin{aligned}
P_n(T) &= \sum_{j=o}^{\min\{n,\, N_R - N_S\}} P_j(o)\binom{T}{n-j} q^{n-j}(1-q)^{T-n+j}, \quad n = o, ---, N_R - 1 \\
P_K(o) &= P_{K+N_S}(T) \qquad\qquad K = 1, ---, N_R - N_S \\
&\sum_{j=o}^{N_R - N_S} P_j(o) = 1 \\
&\sum_{j=o}^{N_R} P_j(T) = 1.
\end{aligned}\right\} \quad (8)$$

Equations (8) can be reduced to $N_R - N_S + 1$ simultaneous equations for $P_j(o)$, $j = o, ---, N_R - N_S$. Once these equations are solved, the abort probabilities can be determined. The abort probability is given by

$$A_r = \frac{1}{qT} \sum_{j=o}^{N_R - N_S} \sum_{K=1}^{T-M+j} K P_j(o) \binom{T}{M-j+K} q^{M-j+K}(1-q)^{T-M+j-K} \quad (9)$$

Equation (9) states that the abort probability is the expected number of aborts divided by the expected number of vehicle arrivals in T seconds. The analytical results were found to compare quite well with the simulation results, especially in the range of abort rates from .1% to 5%. Results given in this paper are based on computer simulations with a main line occupancy rate of 85% of theoretical capacity. Numerical results for main line occupancy rates of 65% were also obtained but these differed only slightly from those with the larger main line flow. The fact that the analytical results based on a fixed cycle time compared so well with the computer simulation indicates that merging conflicts in the front queue are rarely responsible for a vehicle being aborted.

Figure 6 shows that the subdivision of the station and queue areas will affect the abort rate of vehicles entering the station. For example, the 16-slot station shown in Figure 6 has the highest theoretical capacity for $N_S = 5$. However, the abort rates exceed acceptable values at lower flow rates than for $N_S = 4$. For a one percent abort rate, results show that the configuration $N_S = 4$, $N_{12} = 8$, $N_F = 4$ gives the highest throughput.

In Figure 5, stations having sizes from 3 to 30 slots were examined with various subdivisions of station and queue areas. Each point on the graph represents the maximum flow for a 1% and .5% abort rate for a given station size. The results indicate an almost linear relation between total station size in slots and maximum flow rate at constant abort level. One can obtain the following empirical formula for throughput versus total station size from the curve of Figure 5:

$$F_{max} = 39.3N - 98.2 \tag{10}$$

where F_{max} is the maximum flow at a 1% abort rate and N is the number of slots in the station.

Curves of average waiting time versus throughput are plotted in Figure 4. These values represent the total time a vehicle is delayed in the station and queue areas. Therefore, the curves give estimates on the total delay which a passenger would experience at both ends of his trip. With throughput corresponding to low abort rates, average delay times are never more than twice the minimum possible value for a particular station configuration.

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References

BERGMANN, D. R. (1972). Capacity of Automated Fixed Guideway Transportation System Stations with Multiple Loading Positions. Paper presented at 41st Natl. Meeting of ORSA, New Orleans, April, 1972.

BERGREN, L. E. (1971). Off Line Station Operational Characteristics. Proc. 1st Natl. Conf. on Personal Rapid Transit, Minneapolis, Nov. 1-3, 1971.

BUSH, L. R. (1972). The Economics of High Capacity PRT Systems. Personal Rapid Transit - A Collection of Papers on a New Type of Urban Transportation, edited by J. E. Anderson, J. L. Dais, W. L. Garrard, A. L. Kornhauser, pp. 231-250.

DAIS, J. L. (1972). Minichanges, Stations and Geometry in PRT. Personal Rapid Transit.

DAIS, J. L. and KORNHAUSER, A. L. (1973). Economic, Environmental and Design Aspects of Large Scale PRT Networks. Paper accepted for publication by Highway Research Board.

DEMAG FORDERTECHNIK, Wetter a.d.-Ruhr and MESSERSCHMIDT-BOLKOW-BLOHM, G.m.g.H. Ottobrunn, Munchen, (1971). Systems Analysis, Nahverkehrsmittel Cabinentaxi.

GODFREY, M. B. (1968). Merging in Automated Transportation Systems. Doctor of Science thesis, MIT, June, 1968.

HAMILTON, W. F. and NANCE, D.K. (1969). Systems Analysis of Urban Transportation. Scientific American, Vol. 221, No. 1, pp. 19-21, July.

MUNSON, A. V. JR. (1972). Quasi-synchronous Control of High-Capacity PRT Networks. Personal Rapid Transit.

ROYAL AIRCRAFT ESTABLISHMENT (1969). Assessment of Auto-taxi Urban Transportation, Part 1 and 2. Technical Report 68287.

UNIV. OF MINN. TASK FORCE ON NEW CONCEPTS IN URBAN TRANSPORTATION (1973). Planning for Personal Rapid Transit. Dept. of Mech. Eng., Univ. of Minn.

WILSON, D. G., GODFREY, M. B. and FULLER, H. H., JR. (1971). Dual Mode Vehicle, Terminal, and Network Alternatives for Automated Guideway Transportation. S.A.E. Auto. Engr. Congr. Reprint 710112.

YORK, H. L. (1973). Simulation of a PRT Network Under Quasi-Synchronous Control. To be presented at 2nd International Conference on PRT, Minneapolis, 1973.

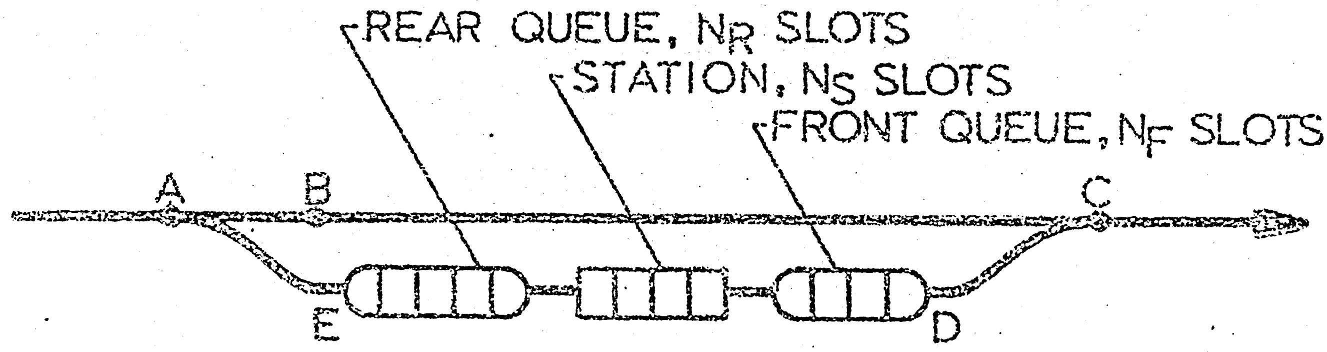
List of Figures

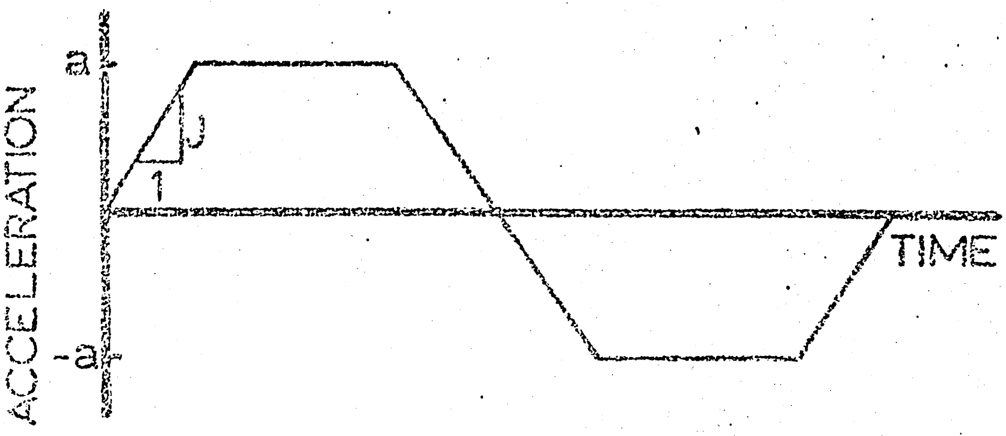
Table 1. Nominal Value of Parameters

Parameter	Assumed Value
Headway Time, T	.5 seconds
Jerk, J	8 ft/sec^3
Acceleration, a	8 ft/sec^2
Vehicle Length, L	16 feet
Station Dwell Time, T_D	15 seconds
Slot Occupancy, p	85%

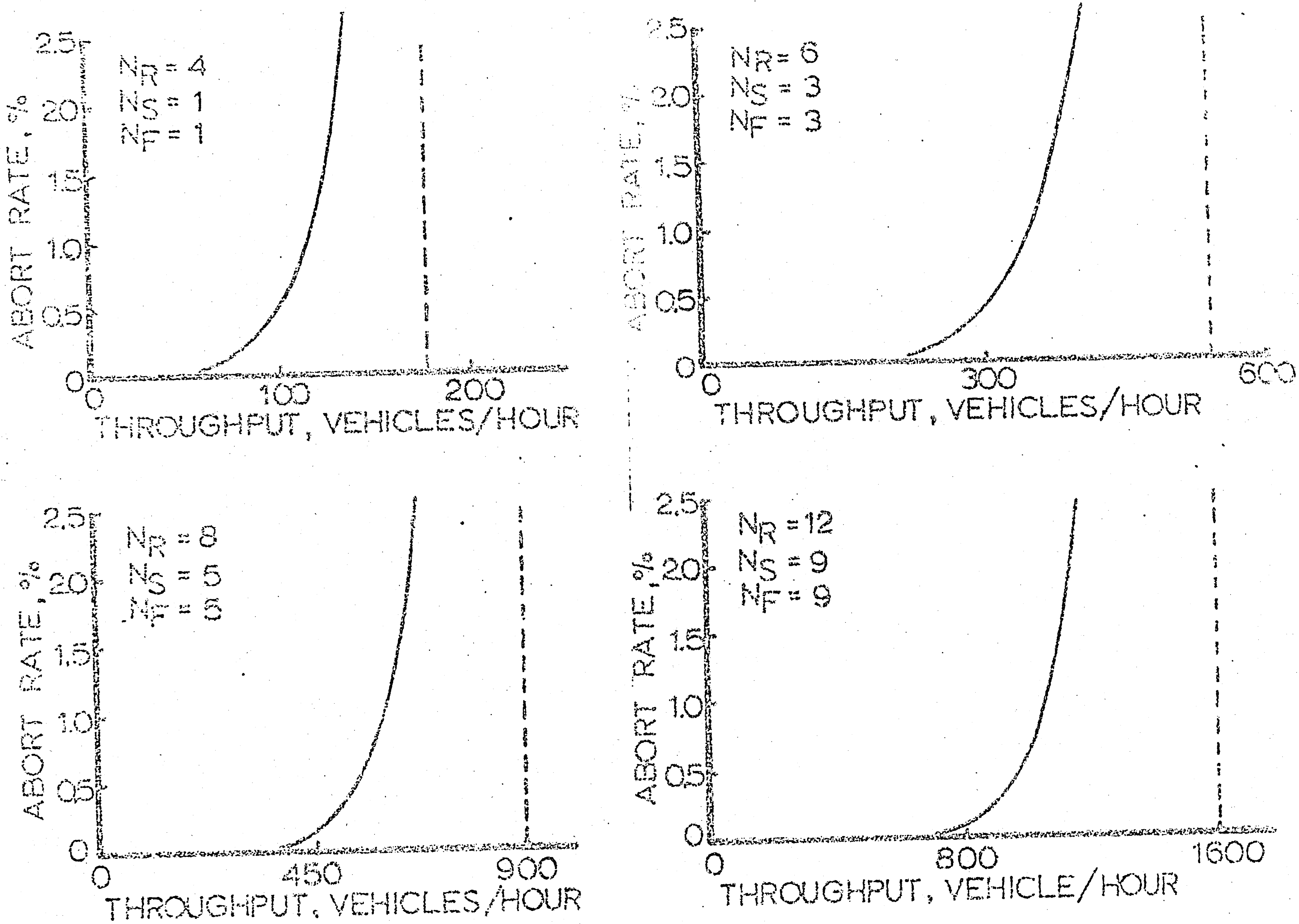
Table 2. Recommended Distribution of Station and Queue Slots

N	=	N_R	+	N_F	+	N_S	N	=	N_R	+	N_F	+	N_S
3		1		1		1	17		9		4		4
4		2		1		1	18		8		5		5
5		3		1		1	19		9		5		5
6		4		1		1	20		10		5		5
7		3		2		2	21		9		6		6
8		4		2		2	22		10		6		6
9		5		2		2	23		11		6		6
10		6		2		2	24		10		7		7
11		5		3		3	25		11		7		7
12		6		3		3	26		12		7		7
13		7		3		3	27		13		7		7
14		6		4		4	28		12		8		8
15		7		4		4	29		13		8		8
16		8		4		4	30		12		9		9

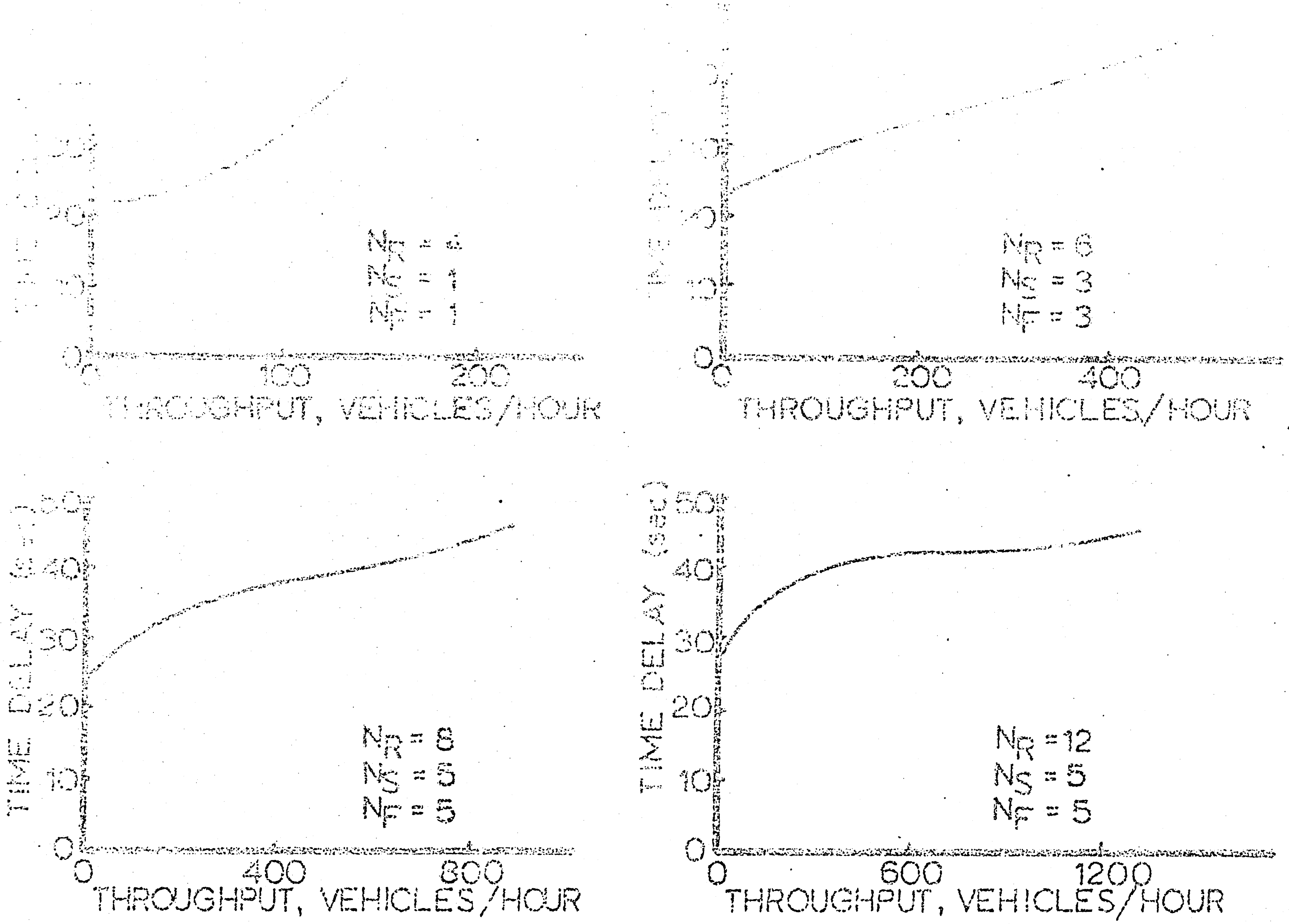

REAR QUEUE, N_R SLOTS
STATION, N_S SLOTS
FRONT QUEUE, N_F SLOTS
A
B
C
D
E


ACCELERATION
a
-a
1
TIME

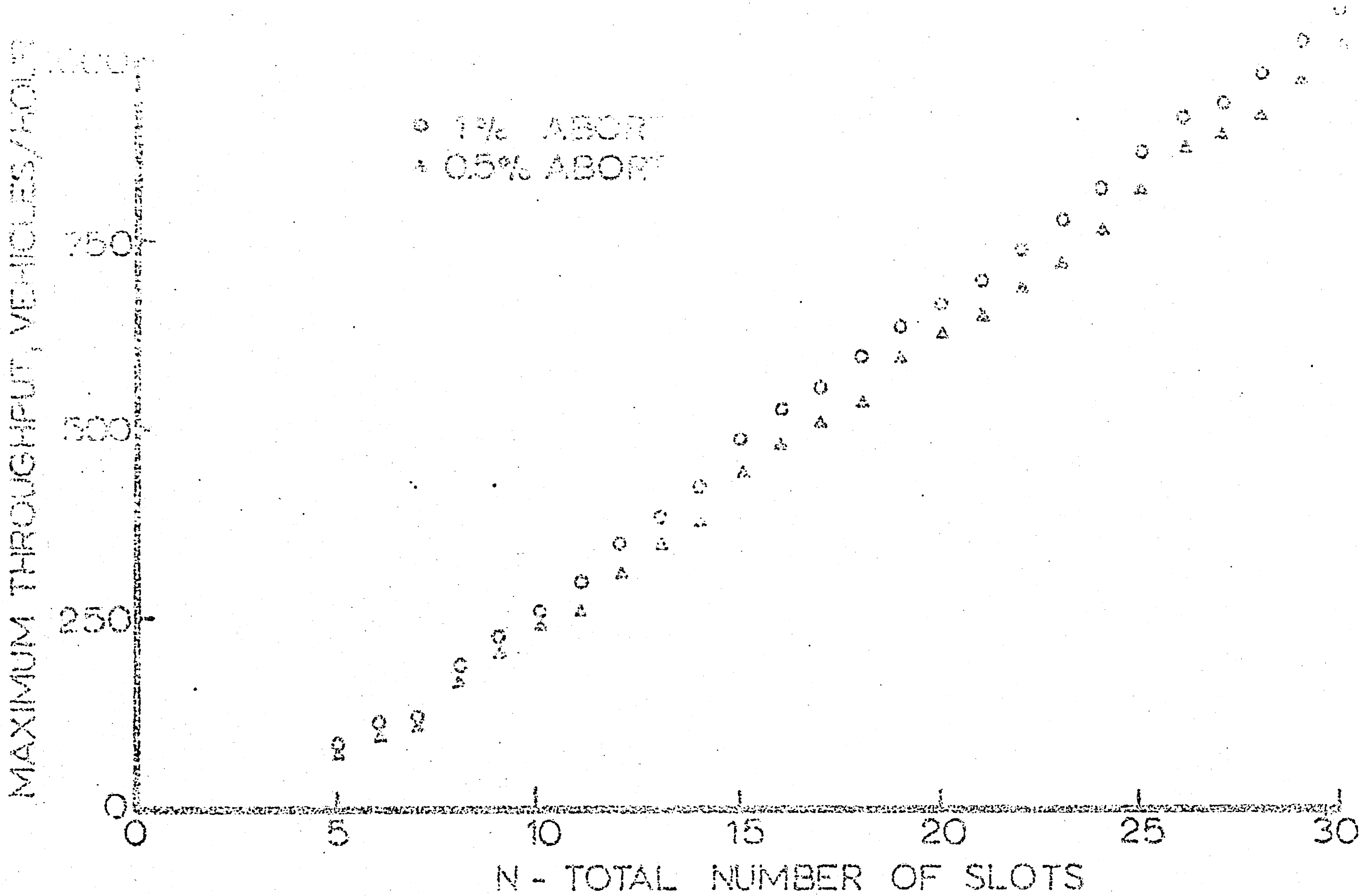
2



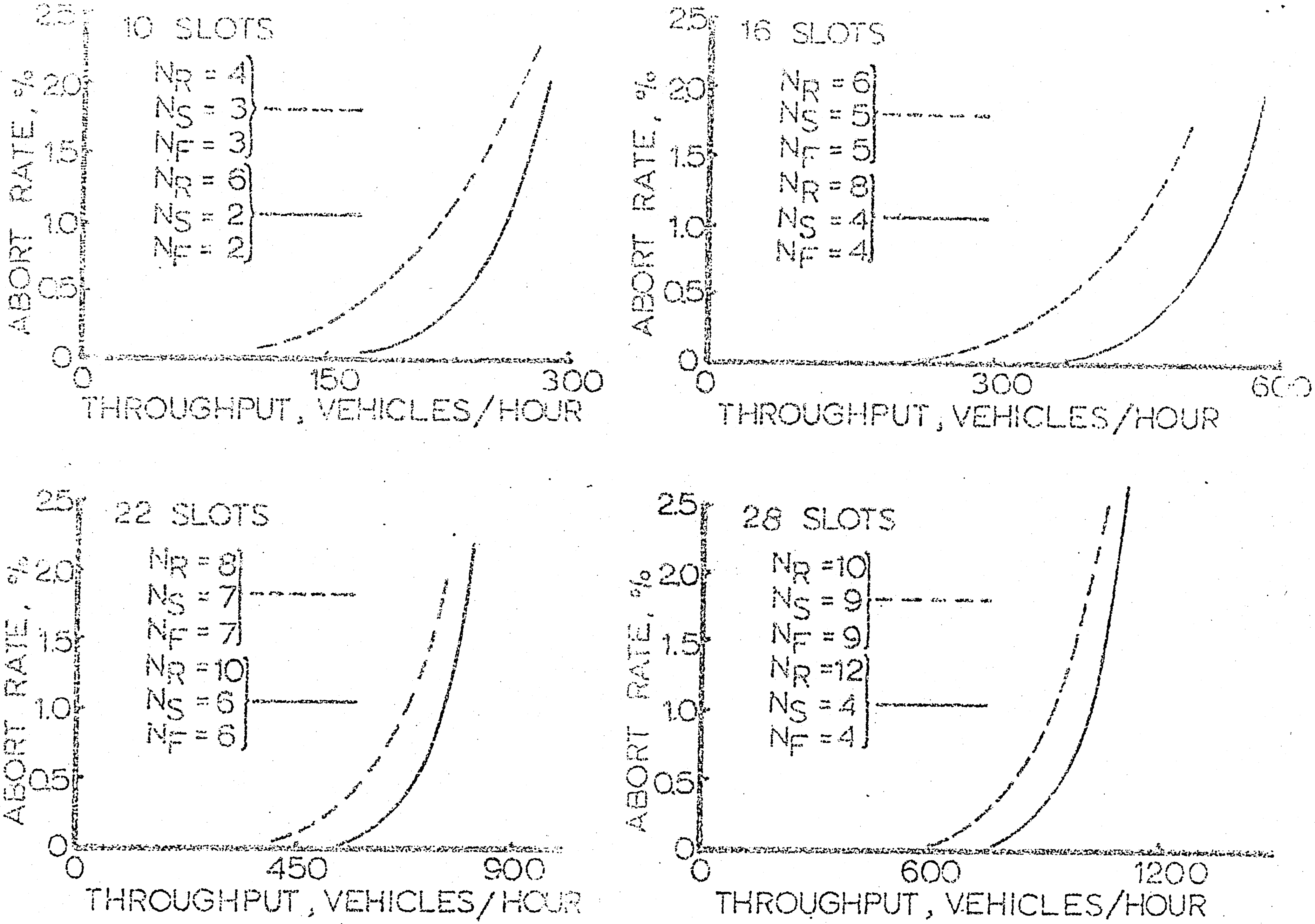
$N_R = 4$
$N_S = 1$
$N_F = 1$
ABORT RATE, %
THROUGHPUT, VEHICLES/HOUR
0
0.5
1.0
1.5
2.0
2.5
100
200
$N_R = 6$
$N_S = 3$
$N_F = 3$
ABORT RATE, %
THROUGHPUT, VEHICLES/HOUR
300
600
$N_R = 8$
$N_S = 5$
$N_F = 5$
ABORT RATE, %
THROUGHPUT, VEHICLES/HOUR
450
900
$N_R = 12$
$N_S = 9$
$N_F = 9$
ABORT RATE, %
THROUGHPUT, VEHICLE/HOUR
800
1600



0
100
200
THROUGHPUT, VEHICLES/HOUR
N_R = 4
N_S = 1
N_F = 1
0
10
20
30
0
200
400
THROUGHPUT, VEHICLES/HOUR
N_R = 6
N_S = 3
N_F = 3
TIME DELAY
0
10
20
30
40
50
0
400
800
THROUGHPUT, VEHICLES/HOUR
N_R = 8
N_S = 5
N_F = 5
TIME DELAY (sec)
0
10
20
30
40
50
0
600
1200
THROUGHPUT, VEHICLES/HOUR
N_R =12
N_S = 5
N_F = 5



1% ABORT
0.5% ABORT
MAXIMUM THROUGHPUT, VEHICLES/HO
750
500
250
0
0
5
10
15
20
25
30
N - TOTAL NUMBER OF SLOTS


10 SLOTS
ABORT RATE, %
2.5
2.0
1.5
1.0
0.5
0
$N_R = 4$
$N_S = 3$
$N_F = 3$
$N_R = 6$
$N_S = 2$
$N_F = 2$
0
150
300
THROUGHPUT, VEHICLES/HOUR
16 SLOTS
ABORT RATE, %
2.5
2.0
1.5
1.0
0.5
0
$N_R = 6$
$N_S = 5$
$N_F = 5$
$N_R = 8$
$N_S = 4$
$N_F = 4$
0
300
600
THROUGHPUT, VEHICLES/HOUR
22 SLOTS
ABORT RATE, %
2.5
2.0
1.5
1.0
0.5
0
$N_R = 8$
$N_S = 7$
$N_F = 7$
$N_R = 10$
$N_S = 6$
$N_F = 6$
0
450
900
THROUGHPUT, VEHICLES/HOUR
28 SLOTS
ABORT RATE, %
2.5
2.0
1.5
1.0
0.5
0
$N_R = 10$
$N_S = 9$
$N_F = 9$
$N_R = 12$
$N_S = 4$
$N_F = 4$
0
600
1200
THROUGHPUT, VEHICLES/HOUR